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A COMPARISON OF FIXED AND VARIABLE TIME OF ARRIVAL NAVIGATION FOR INTERPLANETARY FLIGHT

by

Richard H. Battin

May 1960

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ABSTRACT

Two types of self-contained navigation schemes are contrasted for the case of an unmanned spacecraft launched from Earth and established in a free-fall solar orbit destined to contact either Venus or Mars. A statistical study of the navigation errors and velocity corrections is made for several different trajectories using a three-dimensional model of the Solar System. It is shown that if a certain degree of flexibility is permitted in the arrival time at the destination planet, both the position accuracy and total velocity correction required can be improved by as much as a factor of two.

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A COMPARISON OF FIXED AND VARIABLE TIME
OF ARRIVAL NAVIGATION FOR INTERPLANETARY FLIGHT

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1. Introduction

As the scope of interplanetary ventures broadens, the need for self-sufficiency in spacecraft operations will become apparent. Self-contained navigation systems operating without radiation contact with the Earth will provide the only answer to the problem of spacecraft guidance for all but the most elementary kind of missions.

A general scheme for self-contained interplanetary navigation has been described in Reference 2. The process involves a sequence of velocity corrections at a number of preselected check-points based on deviations in position from a planned trajectory. Position determination is made by on-board optical measurements of angles between lines of sight to various celestial objects and of the apparent angular diameters of planets. The translation of positional errors into required velocity corrections is made by the spacecraft computer. Then, in turn, the micro-rocket propulsion system alters the velocity of the vehicle under direct control of the computer.

A somewhat different navigation theory will be described in this paper and the effectiveness of the two schemes will be contrasted. In the new approach the primary objective will be that of minimizing the fuel consumption of the micro-rocket system without degrading the overall accuracy of the mission. The reduction in fuel requirements is accomplished by permitting the time of contact with the target planet to be a variable chosen in such a way that the velocity correction at any check-point will have the smallest possible magnitude. Just as in the fixed-time-of-arrival navigation scheme, the spaceship is controlled in the vicinity of a reference interplanetary trajectory. When the main propulsion stages of the booster rocket are completed, the vehicle proceeds in a solar orbit with an inaccuracy in the initial velocity attributable to injection guidance errors. At each of a number of check-points during the flight, positional deviations from the precomputed reference trajectory are determined from celestial observations. From these data velocity corrections are computed. By allowing a certain degree of flexibility in the exact time of arrival at the destination planet, only a fraction of the velocity correction needed to direct the vehicle toward the reference arrival point need be applied.

The objective here is first to derive an expression for the appropriate velocity correction in terms of positional deviations from a reference path which is suitable for use by the spaceship. Following this, explicit expressions for the velocity corrections in terms of both the measurement and accelerometer errors are determined. Then, for the purpose of an error analysis of guidance accuracy, the final position and velocity errors are related to the measurement and accelerometer errors.

After the theoretical development a statistical study of the navigation errors and the micro-rocket fuel requirements is made, using a three-dimensional model of the Solar System. For this analysis several different trajectories are subjected to a systematic study to determine the relationship between the total required velocity corrections and the navigational accuracy.

2. The Fundamental Navigation Equation

Let $\underline{r}_s$ (t) and $\underline{v}_s$ (t) denote the position and velocity vectors of the spaceship in an inertial coordinate system with origin at the Sun, and let $\underline{g}(\underline{r}_s, t)$ denote the gravitational acceleration at position $\underline{r}_s$ and time t. Then

$$\frac{d\underline{r}_s}{dt} = \underline{v}_s, \qquad \frac{d\underline{v}_s}{dt} = \underline{g}(\underline{r}_s, t) \tag{1}$$

are the basic equations of motion of the spaceship except for those brief periods during which propulsion is applied.

Let the vectors $\underline{r}_o(t)$ and $\underline{v}_o(t)$ represent the position and velocity at time t associated with the prescribed reference trajectory, and define

$$\delta\underline{r}(t) = \underline{r}_s(t) - \underline{r}_o(t), \quad \delta\underline{v}(t) = \underline{v}_s(t) - \underline{v}_o(t). \tag{2}$$

Then, the deviations $\delta\underline{r}$ and $\delta\underline{v}$ may be approximately related by means of the linearized differential equations:

$$\frac{d(\delta\underline{r})}{dt} = \delta v, \qquad \frac{d(\delta\underline{v})}{dt} = \mathbf{R}_o(\underline{r}_o, t)\,\delta\underline{r}, \tag{3}$$

where $\mathbf{R}_o(\underline{r}_o, t)$ is a matrix whose elements are the partial derivatives of the components of $\underline{g}(\underline{r}_o, t)$ with respect to the components of $\underline{r}_o$.

A particularly useful fundamental set of solutions of Eq (3) may be developed in the following way. Let T_L and T_A be, respectively, the time of launch and the time of arrival at the destination planet. Then, define the matrices $\mathbf{R}(t)$, $\mathbf{R}^*(t)$, $\mathbf{V}(t)$, $\mathbf{V}^*(t)$ as the solutions of the matrix differential equations

$$\begin{aligned} \frac{d\mathbf{R}}{dt} &= \mathbf{V}, & \frac{d\mathbf{R}^*}{dt} &= \mathbf{V}^*, \\ \frac{d\mathbf{V}}{dt} &= \mathbf{R}_o\mathbf{R}, & \frac{d\mathbf{V}^*}{dt} &= \mathbf{R}_o\mathbf{R}^*, \end{aligned} \tag{4}$$

which satisfy the initial conditions

$$\begin{aligned} \mathbf{R}(T_L) &= \mathbf{O}, & \mathbf{R}^*(T_A) &= \mathbf{O}, \\ \mathbf{V}(T_L) &= \mathbf{I}, & \mathbf{V}^*(T_A) &= \mathbf{I}. \end{aligned} \tag{5}$$

Here O and I denote, respectively, the zero and identity matrix. If we now write

$$\delta \underline{r}(t) = \mathbf{R}(t)\,\underline{c} + \mathbf{R}^*(t)\,\underline{c}^*, \tag{6}$$

$$\delta \underline{v}(t) = \mathrm{V}(t)\,\underline{c} + \mathrm{V}^*(t)\,\underline{c}^*, \tag{7}$$

where $\underline{c}$ and $\underline{c}^*$ are arbitrary constant vectors, it follows that these expressions satisfy the perturbation differential equations, Eq (3), and contain precisely the required number of unspecified constants to meet any valid set of initial or boundary conditions.

Assume that measured positional deviations $\delta\underline{\tilde{r}}_{n-1}$ and $\delta\underline{\tilde{r}}_n$ from corresponding reference values are available at the times T_{n-1} and T_n of two successive fixes. Then Eq (6) may be written twice with T_{n-1} and T_n substituted for t. Solving this set for $\underline{c}$ and $\underline{c}^*$ and substituting these values into Eq (7), we have †

$$\delta\underline{\tilde{v}}_n^- = (B_n + B_n^*)\,\delta\underline{\tilde{r}}_n + (\Gamma_n + \Gamma_n^*)\,\delta\underline{\tilde{r}}_{n-1}, \tag{8}$$

where we have used the notation $R_n \equiv R(T_n)$, etc., and defined, for convenience, the matrices

$$\begin{aligned} A_n &= R_{n-1} R_n^{-1}, \qquad C_n = V_n R_n^{-1}, \\ \Gamma_n &= C_n (A_n - A_n^*)^{-1}, \; B_n = -\Gamma_n A_n^*, \end{aligned} \tag{9}$$

with similar definitions for A_n^*, C_n^*, Γ_n^*, $\mathbf{B}_n^*$ obtained from Eq (9) by replacing all starred matrices by the corresponding unstarred ones and conversely.

Eq (8) provides a means of estimating the spaceship velocity at time T_n from positional information at times T_n and

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† The superscripts - and + are used to distinguish the velocity just prior to a correction from the velocity immediately following the correction.

T_{n-1}. For fixed-time-of-arrival navigation there must be added to this velocity a calculated velocity increment $\tilde{\underline{\Delta}}_n$ to arrive at the point $\underline{r}_o\,(T_A)$ at the time T_A. If the spaceship arrives at the reference point from its present position, there will be a velocity deviation $\delta\tilde{\underline{v}}(T_A)$ upon arrival which is related to $\delta\tilde{\underline{r}}_n$ by

$$\delta\tilde{\underline{r}}_n = R_n^* \,\delta\tilde{\underline{v}}(T_A).$$

The corresponding velocity deviation at time T_n,

$$V_n^* \,\delta\tilde{\underline{v}}(T_A) = V_n^* R_n^{*-1} \,\delta\tilde{\underline{r}}_n = C_n^* \,\delta\tilde{\underline{r}}_n,$$

is precisely that which must be established at time T_n. Hence, the fixed-time-of-arrival required velocity correction is given by

$$\tilde{\underline{\Delta}}_n = C_n^* \,\delta\tilde{\underline{r}}_n - \delta\tilde{\underline{v}}_n = H_n \,\delta\tilde{\underline{r}}_n - P_n \,\delta\tilde{\underline{r}}_{n-1}, \tag{10}$$

where the matrices H_n and P_n are defined by

$$H_n = C_n^* - (B_n + B_n^*), \qquad P_n = \Gamma_n + \Gamma_n^* . \tag{11}$$

In order to calculate the variable-time-of-arrival required velocity correction, let us consider the effect of changing the arrival time T_A by a small amount δT. Let $\underline{r}_p(t)$ and $\underline{v}_p(t)$ be, respectively, the position and velocity vectors of the target planet. Then the new point of contact will be $\underline{r}_p(T_A + \delta T)$, and associated therewith will be a somewhat different reference path. Let $\delta\underline{v}_o(T_L)$ be the vector change in launch velocity from the old reference trajectory which is needed to establish the spaceship in the new reference path. From the definition of the **R** matrix, it follows that at time T_A the spaceship position will be

$$\underline{r}_o\,(T_A) + R_A \,\delta\underline{v}_o\,(T_L).$$

At a time δT later the spaceship position will be

$$\underline{r}_o(T_A) + R_A \delta \underline{v}_o(T_L) + \underline{v}_o(T_A)\delta T,$$

and the corresponding planet position will be

$$\underline{r}_p(T_A) + \underline{v}_p(T_A)\,\delta T.$$

Assuming contact to be made at time $T_A + \delta T$, these positions are the same and we may solve for $\delta \underline{v}_o(T_L)$ to obtain

$$\delta \underline{v}_o(T_L) = - R_A^{-1}\,\underline{v}_R(T_A)\,\delta T, \tag{12}$$

where

$$\underline{v}_R(T_A) = \underline{v}_s(T_A) - \underline{v}_p(T_A) \tag{13}$$

is the velocity of the spaceship relative to the planet at the nominal arrival time T_A.

Now at the n^{th} check point, the vector differences in position $\delta \underline{r}_o(T_n)$ and velocity $\delta \underline{v}_o(T_n)$ between the old and new reference trajectories are simply

$$\delta \underline{r}_o(T_n) = R_n \delta \underline{v}_o(T_L), \qquad \delta \underline{v}_o(T_n) = V_n \delta \underline{v}_o(T_L).$$

Hence, from Eq (12)

$$\delta \underline{r}_o(T_n) = - R_n R_A^{-1}\,\underline{v}_R(T_A)\,\delta T, \tag{14}$$

$$\delta \underline{v}_o(T_n) = - V_n R_A^{-1}\,\underline{v}_R(T_A)\,\delta T. \tag{15}$$

Let the measured deviation in position from the old reference path be $\delta \tilde{\underline{r}}_n$, while $\partial \tilde{\underline{r}}_n$ is the corresponding deviation from the new reference path. Then

$$\partial \tilde{\underline{r}}_n = \delta \tilde{\underline{r}}_n - \delta \tilde{\underline{r}}_o\,(T_n). \tag{16}$$

With similar definitions for velocity deviations, we have

$$\partial \tilde{\bar{\underline{v}}}_n = \delta \tilde{\underline{v}}_n - \delta \underline{v}_o(T_n). \tag{17}$$

By following the same arguments which led to Eq (10), we find that the velocity correction $\tilde{\underline{\Delta}}'_n$ (δT) to reach the new point of contact is

$$\tilde{\underline{\Delta}}'_n (\delta T) = C^*_n \, \partial \tilde{\underline{r}}_n - \partial \tilde{\bar{\underline{v}}}_n. \tag{18}$$

Using Eq (16), (17), (14), (15), and (10), we may write this in the form

$$\tilde{\underline{\Delta}}'_n (\delta T) = \tilde{\underline{\Delta}}_n - \underline{\nu}_n \, \delta T, \tag{19}$$

where, for convenience, we have defined the vector $\underline{\nu}_n$ by

$$\underline{\nu}_n = \Lambda_n \, R_A^{-1} \, \underline{v}_R \, (T_A) \tag{20}$$

and the matrix Λ_n by

$$\Lambda_n = V_n - C^*_n \, R_n. \tag{21}$$

With the object of picking δT so as to minimize the magnitude of $\tilde{\underline{\Delta}}'_n$ (δT), clearly the best choice is that which will render $\tilde{\underline{\Delta}}'_n(\delta T)$ normal to $\underline{\nu}_n$. Calling this value $\delta \tilde{T}_A$, we have, from Eq (19),

$$\delta \tilde{T}_A = \frac{\tilde{\underline{\Delta}}_n \cdot \underline{\nu}_n}{\underline{\nu}_n \cdot \underline{\nu}_n} \,. \tag{22}$$

As a consequence, the velocity correction $\tilde{\underline{\Delta}}'_n$ of smallest magnitude which will accomplish the mission is simply related to $\tilde{\underline{\Delta}}_n$ by

$$\tilde{\underline{\Delta}}'_n = M_n \, \tilde{\underline{\Delta}}_n. \tag{23}$$

The matrix M_n is defined by †

$$M_n = I - \underline{\nu}_n \underline{\nu}_n^T / \underline{\nu}_n \cdot \underline{\nu}_n \tag{24}$$

and is a projection operator.

3. Analysis of the Velocity Correction

In order to provide a basis for the selection of check-points, we will derive a relationship which will show explicitly how the velocity correction at time T_n is related to the initial velocity errors at launch, the errors associated with the positional fix, and the errors in establishing the desired velocity corrections at the previous check-points. For this purpose let $\underline{\tilde{\Delta}}_n'$ and $\underline{\eta}_n$ denote, respectively, the actual velocity applied at time T_n and the error made in the application of the desired correction $\underline{\tilde{\Delta}}_n'$. Thus

$$\underline{\tilde{\Delta}}_n' = \underline{\Delta}_n' + \underline{\eta}_n. \tag{25}$$

Similarly, we define $\underline{\epsilon}_n$ and $\underline{\delta}_n$, respectively, as the vector difference between the inferred and actual position and velocity deviations at time T_n, i.e.,

$$\delta\underline{\tilde{r}}_n = \delta\underline{r}_n + \underline{\epsilon}_n, \qquad \delta\underline{\tilde{v}}_n^- = \delta\underline{v}_n^- + \underline{\delta}_n. \tag{26}$$

Therefore, it follows from Eq (25) and (10) that

$$\underline{\Delta}_n' + \underline{\eta}_n = M_n \left[C_n^* (\delta\underline{r}_n + \underline{\epsilon}_n) - (\delta\underline{v}_n^- + \underline{\delta}_n) \right]. \tag{27}$$

However, from Eq (8) and (26) we have

$$\underline{\delta}_n = (B_n + B_n^*) \underline{\epsilon}_n + (\Gamma_n + \Gamma_n^*) \underline{\epsilon}_{n-1}. \tag{28}$$

Thus, Eq (27) may be written as

† The superscript T on a matrix is used to denote the matrix transpose.

$$\underline{\Delta}'_n = M_n(C^*_n\,\delta\underline{r}_n - \delta\underline{v}^-_n) + M_n(H_n\underline{\epsilon}_n - P_n\underline{\epsilon}_{n-1}) - \underline{\eta}_n\,. \tag{29}$$

It now remains to express $C^*_n\,\delta\underline{r}_n - \delta\underline{v}^-_n$ in terms of the error vectors at the present and previous check-points.

To this end we note that at time T^-_n we have

$$C^*_n\,\delta\underline{r}_n - \delta\underline{v}^-_n = -\Lambda_n\underline{c}\,,$$

which is obtained by premultiplying Eq (6) by C^*_n and subtracting Eq (7) with $t = T^-_n$. The constant vector $\underline{c}$ is determined from Eq (6) and (7) with $t = T^*_{n-1}$. We find

$$\underline{c} = -\Lambda^{-1}_{n-1}\,(C^*_{n-1}\,\delta\underline{r}_{n-1} - \delta\underline{v}^*_{n-1})\,. \tag{30}$$

Noting that

$$\delta\underline{v}^+_{n-1} = \delta\underline{v}^-_{n-1} + \underline{\Delta}'_{n-1}\,,$$

we have

$$C^*_n\,\delta\underline{r}_n - \delta\underline{v}^-_n = \Lambda_n\Lambda^{-1}_{n-1}(C^*_{n-1}\delta\underline{r}_{n-1} - \delta\underline{v}^-_{n-1} - \underline{\Delta}'_{n-1})\,. \tag{31}$$

Eq (31) may be used as a recursion formula so that by successive applications we have

$$C^*_n\,\delta\underline{r}_n - \delta\underline{v}^-_n = -\Lambda_n\sum_{k=0}^{n-1}\Lambda^{-1}_k\,\underline{\Delta}'_k\,, \tag{32}$$

if we define

$$\underline{\Delta}'_o = \delta\underline{v}(T_L) = -\,\underline{\eta}_o$$

as the error in the initial launch velocity. Thus, Eq (29) becomes

$$\underline{\Delta}_n' = M_n(H_n \underline{\epsilon}_n - P_n \underline{\epsilon}_{n-1}) - \underline{\eta}_n - M_n \Lambda_n \sum_{k=0}^{n-1} \Lambda_k^{-1} \underline{\Delta}_k', \tag{33}$$

and $\underline{\Delta}_n'$ is expressed recursively in terms of all previous measurement and accelerometer errors.

In order to express $\underline{\Delta}_n'$ directly in terms of the errors, let us consider the following.

Lemma: If a sequence of vectors $\underline{a}_0, \underline{a}_1, \ldots, \underline{a}_n$ is defined by

$$\underline{a}_0 = \underline{b}_0$$

$$\underline{a}_1 = \underline{b}_1 + \psi_1 \Omega_0 \underline{a}_0 \tag{34}$$

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$$\underline{a}_n = \underline{b}_n + \psi_n \sum_{k=0}^{n-1} \Omega_k \underline{a}_k \qquad n \geq 2$$

where $\underline{b}_0, \ldots, \underline{b}_n; \psi_1, \ldots, \psi_n$; and $\Omega_0, \ldots, \Omega_{n-1}$ are arbitrary sequences of vectors and matrices, then

$$\underline{a}_n = \underline{b}_n + \psi_n \Omega_{n-1} \underline{b}_{n-1} + \psi_n \sum_{k=0}^{n-2} \prod_{j=n-1}^{k+1} (I + \Omega_j \psi_j) \Omega_k \underline{b}_k \qquad n \geq 2 \tag{35}$$

Proof: If we define

$$\underline{d}_n = \sum_{k=0}^{n-1} \Omega_k \underline{a}_k \qquad n \geq 2 \tag{36}$$

then we have

$$\underline{a}_n = \underline{b}_n + \psi_n \underline{d}_n$$

and it is sufficient to show that

$$\underline{d}_n = \Omega_{n-1}\underline{b}_{n-1} + \sum_{k=0}^{n-2} \prod_{j=n-1}^{k+1} (I + \Omega_j \psi_j)\, \Omega_k \underline{b}_k \qquad n \geq 2 . \tag{37}$$

For this purpose we note that, from the definitions of $\underline{d}_n$ and $\underline{a}_n$, we have

$$\begin{aligned} \underline{d}_{n+1} &= \underline{d}_n + \Omega_{n-1}\underline{a}_{n-1} \\ &= \underline{d}_n + \Omega_{n-1}(\underline{b}_{n-1} + \psi_{n-1}\underline{d}_n) \\ &= (I + \Omega_{n-1}\psi_{n-1})\, \underline{d}_n + \Omega_{n-1}\underline{b}_{n-1} \end{aligned}$$

as a difference equation for $\underline{d}_n$. Furthermore, it is a simple matter to verify that $\underline{d}_n$, as given by Eq (37), is the solution. For $n = 2$, Eq (37) yields

$$\underline{d}_2 = \Omega_1 \underline{b}_1 + (I + \Omega_1 \psi_1)\, \Omega_0 \underline{b}_0$$

which is clearly the same as that obtained from Eq (36), using the definitions of $\underline{a}_0$ and $\underline{a}_1$. Hence, the lemma is proved.

We may apply the lemma to the problem at hand by making the following identifications:

$$\underline{a}_n \sim \underline{\Delta}_n' , \qquad \psi_n \sim - M_n \Lambda_n ,$$

$$\underline{b}_n \sim M_n (H_n \underline{\epsilon}_n - P_n \underline{\epsilon}_{n-1}) - \underline{\eta}_n , \quad \Omega_k \sim \Lambda_k^{-1} .$$

Then if we define

$$X_{k,n} = \begin{cases} I & \text{for } k = n-1 \\ \displaystyle\prod_{j=n-1}^{k+1} (I - \Lambda_j^{-1} M_j \Lambda_j) & \text{for } k \leq n-2 \end{cases} \tag{38}$$

we may use Eq (35 to write Eq (33) in the form

$$\underline{\Delta}_n' = M_n (H_n \underline{\epsilon}_n - P_n \underline{\epsilon}_{n-1}) - \underline{\eta}_n$$

$$- M_n \Lambda_n \sum_{k=0}^{n-1} X_{k,n} \Lambda_k^{-1} \left[M_k (H_k \underline{\epsilon}_k - P_k \underline{\epsilon}_{k-1}) - \underline{\eta}_k \right]. \qquad (39)$$

In the summation indicated in Eq (39), all terms for $k \leq n - 2$ have as a factor

$$M_n \Lambda_n (I - \Lambda_{n-1}^{-1} M_{n-1} \Lambda_{n-1}).$$

Using Eq (20) and (24), this factor may be written as

$$M_n \Lambda_n \Lambda_{n-1}^{-1} \underline{\nu}_{n-1} \underline{\nu}_{n-1}^T \Lambda_{n-1} / \underline{\nu}_{n-1} \cdot \underline{\nu}_{n-1}$$

$$= M_n \Lambda_n \Lambda_{n-1}^{-1} \Lambda_{n-1} R_A^{-1} \underline{v}_R (T_A) \underline{v}_R^T (T_A) R_A^{T-1} \Lambda_{n-1}^T \Lambda_{n-1} / \underline{\nu}_{n-1} \cdot \underline{\nu}_{n-1}$$

$$= M_n (\Lambda_n R_A^{-1} \underline{v}_R (T_A) \underline{v}_R^T (T_A) R_A^{T-1} \Lambda_n^T) \Lambda_n^{T-1} \Lambda_{n-1}^T \Lambda_{n-1} / \underline{\nu}_{n-1} \cdot \underline{\nu}_{n-1}$$

$$= M_n \underline{\nu}_n \underline{\nu}_n^T \Lambda_n^{T-1} \Lambda_{n-1}^T \Lambda_{n-1} / \underline{\nu}_{n-1} \cdot \underline{\nu}_{n-1}$$

$$= M_n (I - M_n) \Lambda_n^{T-1} \Lambda_{n-1}^T \Lambda_{n-1} \left(\frac{\underline{\nu}_n \cdot \underline{\nu}_n}{\underline{\nu}_{n-1} \cdot \underline{\nu}_{n-1}} \right).$$

Now, since M_n is a projection operator it follows that

$$M_n (I - M_n) = O.$$

Hence, all terms in Eq (39) for $k \leq n - 2$ are identically zero, and we may write

$$\underline{\Delta}_n' = M_n H_n \underline{\epsilon}_n - (M_n P_n + M_n \Lambda_n \Lambda_{n-1}^{-1} M_{n-1} H_{n-1}) \underline{\epsilon}_{n-1}$$

$$+ M_n \Lambda_n \Lambda_{n-1}^{-1} M_{n-1} P_{n-1} \underline{\epsilon}_{n-2} - \underline{\eta}_n + M_n \Lambda_n \Lambda_{n-1}^{-1} \underline{\eta}_{n-1} . \qquad (40)$$

This relationship expresses explicitly the velocity impulse actually applied at time T_n in terms of present and previous measurement errors as well as errors in controlling the applied velocity. It is interesting to note that the corresponding expression for fixed-time-of-arrival navigation is obtained from Eq (40) by replacing the matrices M_n and M_{n-1} by the identity matrix.

4. Navigation Error Analysis

The effects of an initial velocity error, together with imperfect velocity corrections applied at the various check-points, will be: (1) a velocity deviation from the reference value at the destination planet; (2) a positional error or miss distance; and (3) a change from the scheduled arrival time. For use in the statistical analysis of guidance accuracy, we shall now develop appropriate expressions for each of these quantities in terms of the various measurement and accelerometer errors.

a) Velocity Deviation

We write Eq (6) and (7) for $t = T_n$ with $\delta \underline{r}(T_n) = 0$ and $\delta \underline{v}(T_n) = \underline{\Delta}_n'$, and solve for $\underline{c}$ and $\underline{c}^*$ to obtain

$$\underline{c} = \Lambda_n^{-1} \underline{\Delta}_n' , \qquad \underline{c}^* \equiv \Lambda_n^{*-1} \underline{\Delta}_n' .$$

Then, if $\delta \underline{v}_n(T_A)$ is the velocity deviation at time T_A due to the velocity impulse at time T_n, it follows from Eq (7) that

$$\delta \underline{v}_n(T_A) = V(T_A) \underline{c} + \underline{c}^* .$$

(Note that $V^*(T_A)$ is the identity matrix.) Now assuming N check-points, the total effect, obtained by superposition, is expressible as

$$\delta \underline{v}(T_A) = \sum_{n=0}^{N} \left[V(T_A) - R_n^{*-1} R_n \right] \Lambda_n^{-1} \underline{\Delta}_n' , \tag{41}$$

where $\underline{\Delta}_0'$ is the initial velocity error at launch. By means of Eq (40), $\delta \underline{v}(T_A)$ may be expressed in terms of the errors $\underline{\epsilon}$ and $\underline{\eta}$.

b) Positional Error

In order to determine the positional deviation at the time of arrival, we use Eq (6) and (30) in the form

$$\delta \underline{r}(T_A) = R_A \Lambda_N^{-1} \left(\delta \underline{v}_N^- - C_N^* \delta \underline{r}_N + \underline{\Delta}_N' \right) . \tag{42}$$

Then by substituting from Eq (32) with n = N, we obtain

$$\delta \underline{r}(T_A) = R_A \sum_{k=0}^{N} \Lambda_k^{-1} \underline{\Delta}_k' . \tag{43}$$

Only the component of $\delta \underline{r}(T_A)$ perpendicular to the direction of relative motion between the vehicle and the target planet is of interest in determining the actual miss distance. The other component along the direction of motion is more nearly responsible for an error in the scheduled arrival time. Denoting the actual miss distance vector by $\delta \underline{r}_a$, we have

$$\delta \underline{r}_a = M_a \, \delta \underline{r}(T_A), \tag{44}$$

where the matrix M_a is a projection operator

$$M_a = I - \underline{v}_R(T_A) \, \underline{v}_R^T(T_A) \, / \, \underline{v}_R(T_A) \cdot \underline{v}_R(T_A) . \tag{45}$$

It would seem at first that the miss distance at the target planet is a function of measurement and velocity correction errors at all of the previous check-points. Indeed, as can be seen from Eq (43), the positional deviation from the reference arrival point does depend on all past errors; but only the measurement errors at the last two check-points, together with the last accelerometer error, affect the component $\delta \underline{r}_a$. For the proof we use Eq (33) with n = N to write Eq (43) in the form

$$\delta \underline{r}(T_A) = R_A \Lambda_N^{-1} M_n(H_N \underline{\epsilon}_N - P_N \underline{\epsilon}_{N-1}) - R_A \Lambda_N^{-1} \underline{\eta}_N$$

$$+ R_A (I - \Lambda_N^{1} M_N \Lambda_N) \sum_{k=0}^{N-1} \Lambda_k^{-1} \underline{\Delta}'_k . \tag{46}$$

When we apply the projection operator M_a to this vector, the coefficient of the indicated summation can be shown to vanish identically. For if we use the definitions (20) and (24), we have

$$M_a R_A (I - \Lambda_N^{-1} M_N \Lambda_N)$$

$$= M_a R_A \Lambda_N^{-1} \underline{\nu}_N \underline{\nu}_N^T \Lambda_N / \underline{\nu}_N \cdot \underline{\nu}_N$$

$$= M_a R_A \Lambda_N^{-1} \Lambda_N R_A^{-1} \underline{v}_R(T_A) \underline{v}_R^T (T_A) R_A^{T-1} \Lambda_N^{T-1} \Lambda_N / \underline{\nu}_N \cdot \underline{\nu}_N$$

$$= M_a (I - M_a) R_A^{T-1} \Lambda_N^{T-1} \Lambda_N \left(\frac{\underline{v}_R(T_A) \cdot \underline{v}_R(T_A)}{\underline{\nu}_N \cdot \underline{\nu}_N} \right),$$

and from the definition (45) of M_a it follows that

$$M_a (I - M_a) = O$$

Thus, the appropriate expression for the miss distance at the target planet is simply

$$\delta \underline{r}_a = M_a R_A \Lambda_N^{-1} \left[M_N (H_N \underline{\epsilon}_N - P_N \underline{\epsilon}_{N-1}) - \underline{\eta}_N \right]. \qquad (47)$$

c) Change in the Scheduled Time of Arrival

From Eq (22) it is seen that at each check-point the optimum difference in arrival time from the nominal value T_A depends on the velocity correction $\tilde{\underline{\Delta}}_n$ which would be required to carry the vehicle to the nominal point of arrival $\underline{r}_p(T_A)$. If, at each of the previous check-points, the corrections have been of the variable time of arrival type, then $\tilde{\underline{\Delta}}_n$ will be a function of all previous measurement and velocity correction errors. The precise relationship is obtained as follows.

From Eq (10) and (26) we have

$$\tilde{\underline{\Delta}}_n = (C_n^* \delta \underline{r}_n - \delta \underline{v}_n^-) + H_n \underline{\epsilon}_n - P_n \underline{\epsilon}_{n-1},$$

and substituting from Eq (32) gives

$$\tilde{\underline{\Delta}}_n = H_n \underline{\epsilon}_n - P_n \underline{\epsilon}_{n-1} - \Lambda_n \sum_{k=0}^{n-1} \Lambda_k^{-1} \underline{\Delta}_k', \qquad (48)$$

where $\underline{\Delta}_k'$ is expressed in terms of the $\underline{\epsilon}$'s and $\underline{\eta}$'s according to Eq (40). The final indicated change in the scheduled arrival time is obtained from Eq (22) with n = N. We find

$$\delta \tilde{T}_A = (\underline{\nu}_N \cdot \underline{\nu}_N)^{-1} \left\{ \underline{\nu}_N^T \left[H_N \underline{\epsilon}_N - P_N \underline{\epsilon}_{N-1} - \Lambda_N \sum_{k=0}^{N-1} \Lambda_k^{-1} \underline{\Delta}_k' \right] \right\}. \qquad (49)$$

5. Numerical Results and Conclusions

Four trajectories were selected for use as samples in analyzing the fixed and variable-time-of-arrival navigation schemes. These trajectories, which were determined using the methods described in Reference 1, are illustrated in Fig. 1 through 4 and their basic characteristics are summarized in Table 1.

Each of these trajectories is attainable from a circular coasting orbit from Canaveral. In the table is given the launch azimuth from Canaveral together with the latitude and longitude on the Earth's surface at which injection into orbit is to occur. The illustrations show the orbits of the spacecraft and the planets Venus, Earth, and Mars. The paths are shown as solid lines when the orbital plane is above the plane of the ecliptic and broken lines when below. The launch and arrival positions are marked with the corresponding dates. The configuration of the spacecraft and the planets is shown for one instant of time during midcourse on the date indicated in the figures. A shaded circle is used to show the position of the Earth at the time of contact with the target planet.

The method of analysis closely parallels the approach taken in Reference 2 and is entirely statistical in nature. A number of check-points is postulated at which positional deviations from the reference path are determined from celestial observations of the type described in Reference 2.

For our present study, in order to increase the attainable accuracy in the determination of spacecraft position, the number of admissible celestial objects was enlarged and the strategies by which pairs of them could be selected were generalized. The Moon was added to the collection of observable objects within the Solar System and the number of available stars was increased to ten. In order of brightness those chosen are as follows:

	Magnitude		Magnitude
Sirius	-1.58	Arcturus	0.24
Canopus	-0.86	Rigel	0.34
Alpha Centauri	0.06	Procyon	0.48
Vega	0.14	Achernar	0.60
Capella	0.21	Beta Centauri	0.86

At each instant of time along the sample trajectories various combinations of celestial measurements were considered in an effort to reduce the uncertainty in spacecraft position. The standard deviation of the measurement errors was assumed to be 0.05 milliradians or 10.3 seconds of arc and the clock was assumed to drift at a constant rms rate of one part in 100,000. The best obtained rms position and time errors as a function of time from launch are presented in Tables 2 through 5.†

In order to test the concept of variable-time-of-arrival navigation, a number of complete statistical simulations were made using different combinations of times for velocity corrections. The postulated guidance errors were

(1) an RMS injection velocity error of 40 feet per second and

(2) an RMS error in applying any desired velocity correction of 1%.

The injection velocity error corresponds to burn-out of the main propulsion. Therefore, it is necessary to apply a magnification factor of $\left[1 + (v_{esc} / v_R)^2\right]$ to the mean-squared injection velocity error to obtain the mean-squared velocity error after escape. Here v_{esc} and v_R are, respectively, the escape velocity and the excess hyperbolic velocity of the spacecraft.

For the clock error it was assumed that between the times of two consecutive fixes the clock is drifting at a constant rate, where the rate is random, and statistically independent of any previous drift.

In each of the navigation simulations four separate fixes and associated velocity corrections were made. The times selected as check-points were chosen in the following way for

† The notation NO 2ND PLANET in the Tables indicates that the line-of-sight to only one planet was more than 15 degrees away from the Sun-line. Thus, a fix strategy involving more than one planet is not possible if 15 degrees is used as the threshold of visibility.

each trajectory. From the possible times of fix, as listed in Tables 2 through 5, four subsets of times were picked. Then the fix times for each navigation run were selected, one from each group, by a random choice.

The result of each run is represented by a point in the Fig. 5 through 8 where the final position error in miles has been plotted against total velocity correction in feet per second. The envelope of these points is shown in the figures and may be used as one of the principle criteria in planning a mission. This curve expresses the ultimate precision attainable for the trajectory as far as optimizing the miss distance with respect to total velocity correction. The detailed history of each navigation run lying along the envelope curve is presented in Tables 6 through 9.

Mars trajectory I is superior to II with respect to navigation accuracy. One can perhaps correlate this result with the fact that the velocity relative to Mars at arrival is much less for I than for II. The same thesis is borne out when Venus trajectory III is compared with IV.

The distribution of points in Fig. 7 for Venus trajectory III is so widely scattered that it is impossible to recognize any envelope curve from the available data.

As a result of this study the conclusions are immediate. For either navigation scheme it is clear that, in general, position accuracy can be improved only at the expense of extra fuel. Furthermore, the superiority of the variable-time-of-arrival navigation scheme is more than two-fold with regard to both position accuracy and total velocity correction required. For a one-way planetary mission, its advantages seem far to over-balance any potential difficulties which could result from an uncertainty in the exact time of rendezvous with the destination planet.

TRAJECTORY DATA

	EARTH TO MARS		EARTH TO VENUS	
	I	II	III	IV
TIME OF DEPARTURE	NOV. 5,1964	NOV.24,1964	APR.19,1964	APR.19,1964
TIME OF FLIGHT (YEARS)	0.85	0.50	0.45	0.30
INJECTION VELOCITY (FT/SEC)	37484	38583	37410	38826
HYPERBOLIC VELOCITY EXCESS AT EARTH (FT/SEC)	9968	13526	9688	14206
COMPONENTS OF HYPERBOLIC VELOCITY EXCESS IN THE ECLIPTIC COORDINATE SYSTEM (FT/SEC)	-8478 4288 3016	-12788 3633 2493	-2582 9324 510	3678 11471 -7529
SEMI-MAJOR AXIS (A.U.)	1.24466	1.40745	0.84580	0.87093
ECCENTRICITY	0.20788	0.30040	0.18806	0.17330
HYPERBOLIC VELOCITY EXCESS AT DESTINATION PLANET (FT/SEC)	9135	25261	18357	13339
DISTANCE FROM EARTH AT TIME OF CONTACT (A.U.)	1.79539	1.07767	0.95173	0.53476
LAUNCH AZIMUTH FROM CAPE CANAVERAL (DEG)	100	110	100	100
LONGITUDE OF INJECTION POINT (DEG)	125 E	144 E	128 E	1 E
LATITUDE OF INJECTION POINT (DEG)	16 S	5 N	15 S	10 S

TABLE 1

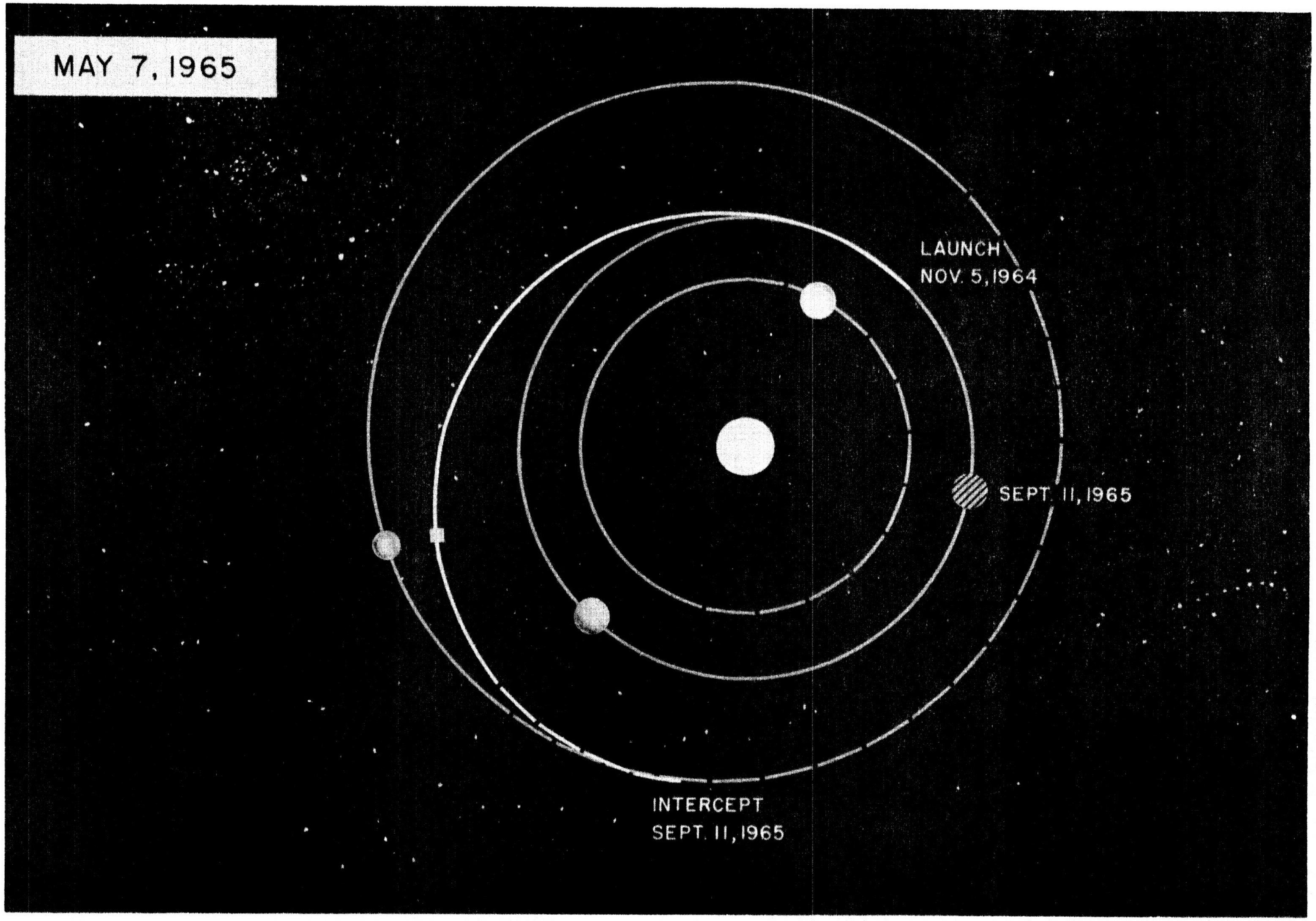

Fig. 1 Mars trajectory I

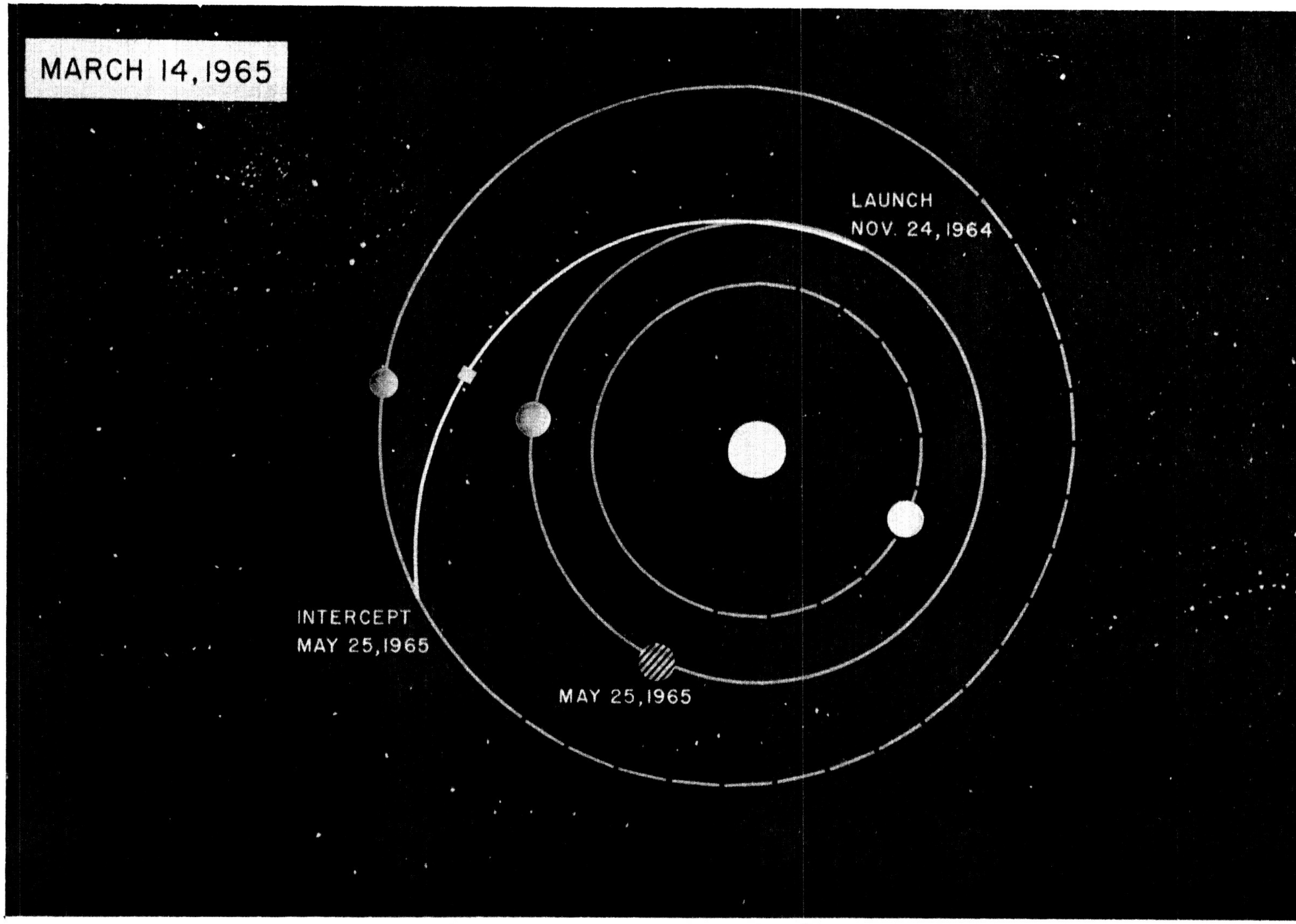

Fig. 2 Mars trajectory II

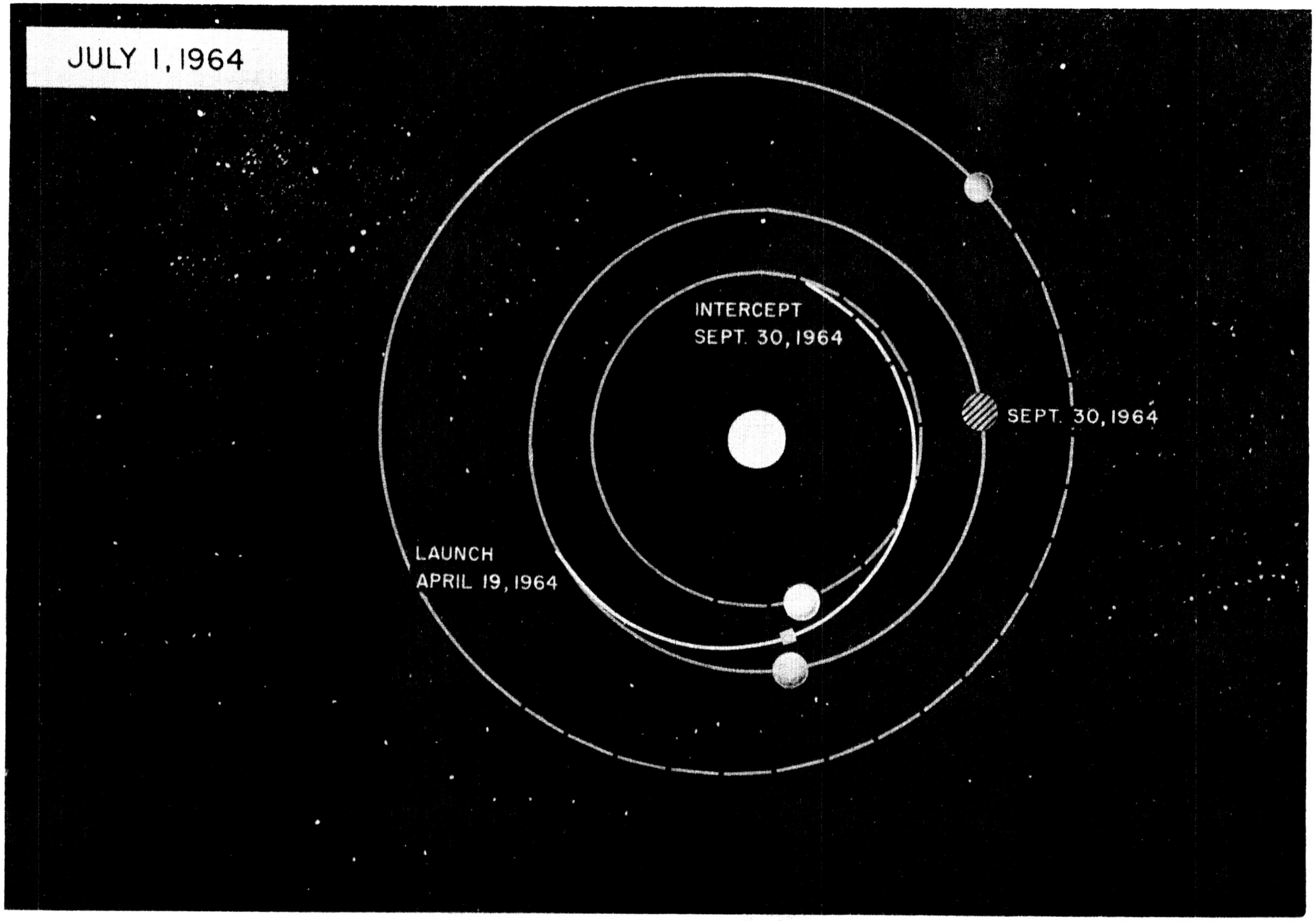

Fig. 3 Venus trajectory III

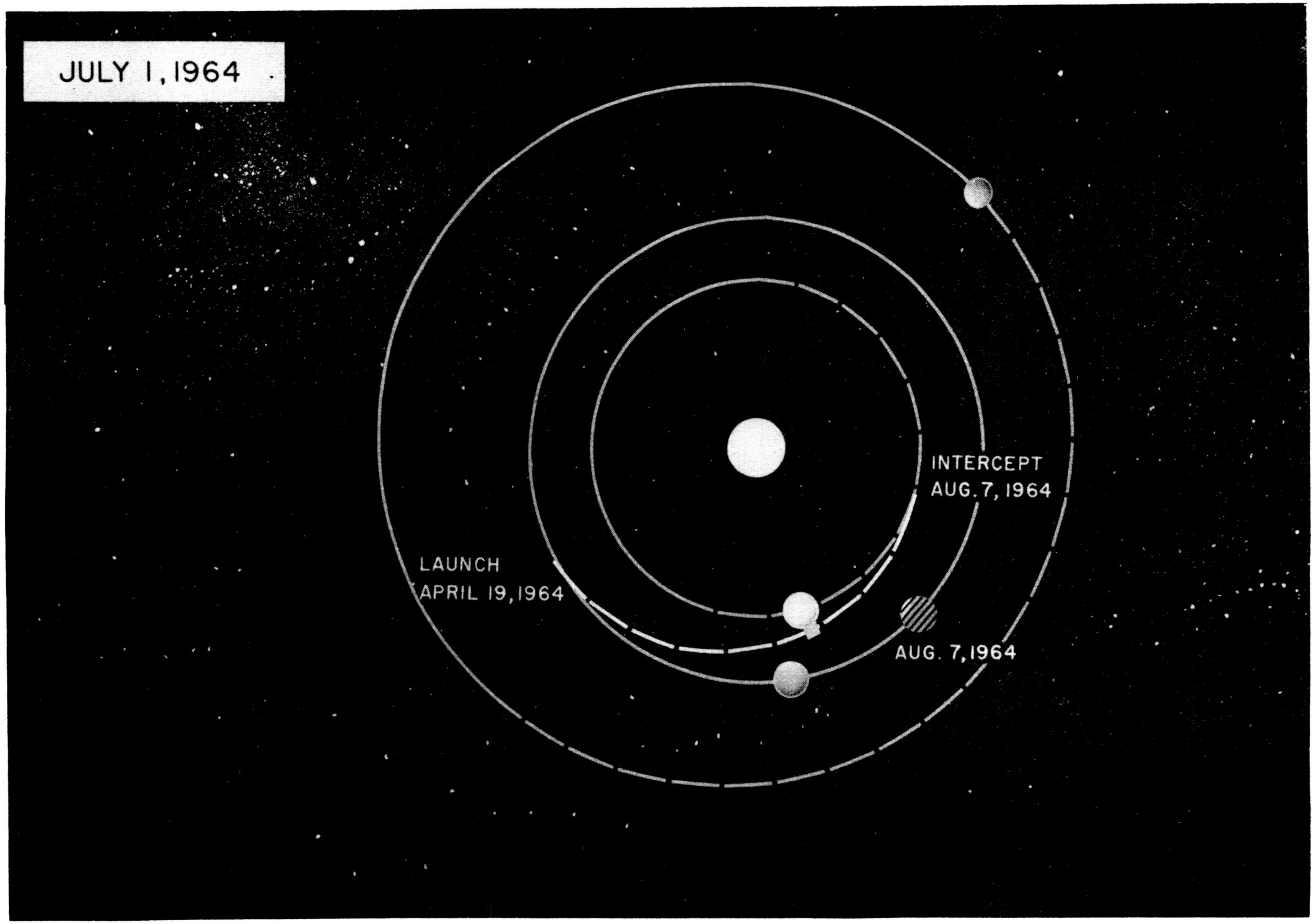

Fig. 4 Venus trajectory IV

CELESTIAL FIX POSITION AND TIME ERRORS

MARS TRAJECTORY NOV.5,1964

V_{RE} = 9968 FT/SEC V_{RM} = 9135 FT/SEC

TIME IN YEARS	RMS POS. ERROR MILES	RMS TIME ERROR HOURS	TIME IN YEARS	RMS POS. ERROR MILES	RMS TIME ERROR HOURS
0.001	10	0.0000	0.425	2926	0.0359
0.002	14	0.0002	0.450	3267	0.0380
0.003	20	0.0003	0.475	3742	0.0401
0.004	28	0.0004	0.500	4233	0.0419
0.005	39	0.0004	0.525	4950	0.0436
0.006	51	0.0005	0.550	5802	0.0452
0.007	67	0.0006	0.575	6689	0.0464
0.008	87	0.0007	0.600	6796	0.0500
0.009	112	0.0008	0.625	6812	0.0520
0.010	143	0.0009	0.650	6733	0.0543
0.025	1292	0.0022	0.675	5854	0.0586
0.050	2430	0.0044	0.700	5081	0.0594
0.075	2899	0.0066	0.725	4396	0.0589
0.100	3453	0.0088	0.750	3885	0.0571
0.125	4026	0.0109	0.775	3612	0.0545
0.150	3757	0.0129	0.800	6137	0.0670
0.175	6414	0.0151	0.825	6049	0.0689
0.200	5049	0.0170	0.840	2890	0.0617
0.225	7644	0.0200	0.841	2422	0.0607
0.250	NO 2ND PLANET		0.842	1964	0.0600
0.275	NO 2ND PLANET		0.843	1537	0.0594
0.300	2400	0.0254	0.844	1156	0.0590
0.325	2339	0.0274	0.845	837	0.0588
0.350	2382	0.0295	0.846	596	0.0587
0.375	2486	0.0316	0.847	444	0.0587
0.400	2633	0.0338	0.848	379	0.0586

TABLE 2

CELESTIAL FIX POSITION AND TIME ERRORS

MARS TRAJECTORY NOV.24,1964

V_{RE} = 13526 FT/SEC V_{RM} = 25261 FT/SEC

TIME IN YEARS	RMS POS. ERROR MILES	RMS TIME ERROR HOURS	TIME IN YEARS	RMS POS. ERROR MILES	RMS TIME ERROR HOURS
0.001	10	0.0001	0.250	NO 2ND PLANET	
0.002	9	0.0002	0.275	2719	0.0234
0.003	11	0.0003	0.300	2634	0.0255
0.004	21	0.0004	0.325	2799	0.0277
0.005	49	0.0004	0.350	2999	0.0298
0.006	119	0.0005	0.375	3326	0.0320
0.007	318	0.0006	0.400	3772	0.0341
0.008	564	0.0007	0.425	4373	0.0365
0.009	450	0.0008	0.450	5107	0.0382
0.010	376	0.0009	0.475	6019	0.0400
0.025	764	0.0022	0.490	6517	0.0420
0.050	3407	0.0044	0.491	6463	0.0421
0.075	3289	0.0066	0.492	6345	0.0421
0.100	3434	0.0087	0.493	6112	0.0422
0.125	4899	0.0109	0.494	5679	0.0423
0.150	6800	0.0130	0.495	4931	0.0423
0.175	6138	0.0149	0.496	3808	0.0424
0.200	8097	0.0174	0.497	2475	0.4248
0.225	NO 2ND PLANET		0.498	1318	0.0408

TABLE 3

CELESTIAL FIX POSITION AND TIME ERRORS

VENUS TRAJECTORY APRIL 19,1964

V_{RE} = 9688 FT/SEC V_{RV} = 18357 FT/SEC

TIME IN YEARS	RMS POS. ERROR MILES	RMS TIME ERROR HOURS	TIME IN YEARS	RMS POS. ERROR MILES	RMS TIME ERROR HOURS
0.001	19	0.0001	0.225	1870	0.0190
0.002	10	0.0002	0.250	1765	0.0208
0.003	9	0.0003	0.275	1793	0.0224
0.004	12	0.0004	0.300	1865	0.0238
0.005	17	0.0004	0.325	1962	0.0249
0.006	24	0.0005	0.350	3650	0.0285
0.007	33	0.0006	0.375	2939	0.0259
0.008	44	0.0007	0.400	2354	0.0277
0.009	55	0.0008	0.425	2143	0.0239
0.010	68	0.0009	0.440	2857	0.0329
0.025	604	0.0022	0.441	2902	0.0329
0.050	1990	0.0044	0.442	2856	0.0330
0.075	4337	0.0066	0.443	2729	0.0330
0.100	5531	0.0088	0.444	2486	0.0328
0.125	7301	0.0109	0.445	2103	0.0325
0.150	12768	0.0131	0.446	1607	0.0321
0.175	21517	0.0149	0.447	1090	0.0317
0.200	2272	0.0170	0.448	668	0.0300

TABLE 4

CELESTIAL FIX POSITION AND TIME ERRORS

VENUS TRAJECTORY APRIL 19,1964

V_{RE} = 14206 FT/SEC V_{RV} = 13339 FT/SEC

TIME IN YEARS	RMS POS. ERROR MILES	RMS TIME ERROR HOURS	TIME IN YEARS	RMS POS. ERROR MILES	RMS TIME ERROR HOURS
0.001	10	0.0001	0.150	1100	0.0129
0.002	14	0.0002	0.175	1026	0.0150
0.003	22	0.0003	0.200	1053	0.0170
0.004	34	0.0004	0.225	1343	0.0188
0.005	49	0.0004	0.250	1875	0.0203
0.006	68	0.0005	0.275	2596	0.0219
0.007	91	0.0006	0.290	2440	0.0241
0.008	116	0.0007	0.291	2266	0.0241
0.009	145	0.0008	0.292	2030	0.0241
0.010	177	0.0009	0.293	1733	0.0241
0.025	1675	0.0022	0.294	1389	0.0240
0.050	3817	0.0044	0.295	1032	0.0240
0.075	1983	0.0066	0.296	701	0.0239
0.100	1545	0.0087	0.297	439	0.0240
0.125	1277	0.0109	0.298	277	0.0241

TABLE 5

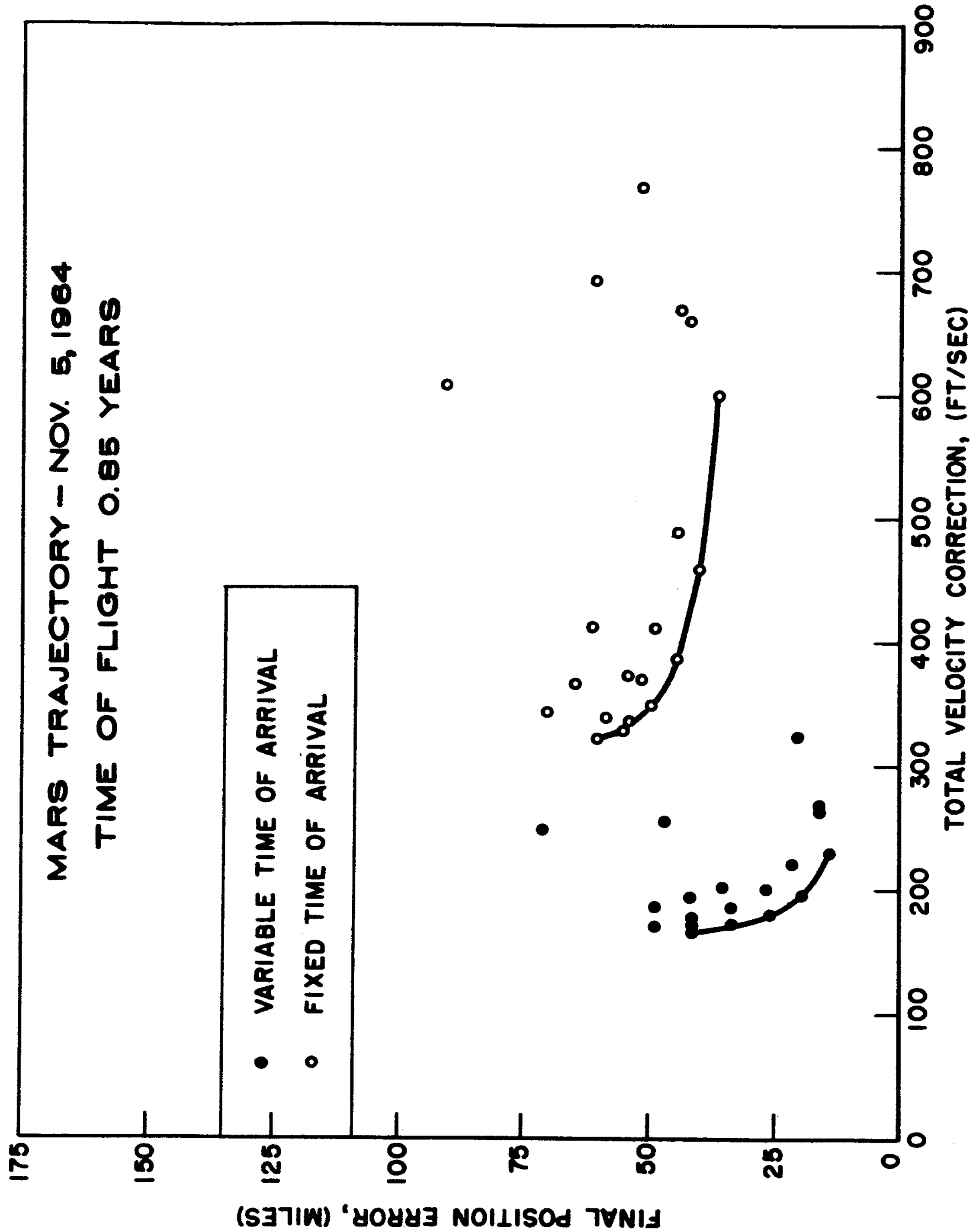
MARS TRAJECTORY – NOV. 5, 1964
TIME OF FLIGHT 0.85 YEARS
VARIABLE TIME OF ARRIVAL
FIXED TIME OF ARRIVAL
FINAL POSITION ERROR, (MILES)
TOTAL VELOCITY CORRECTION, (FT/SEC)
0
25
50
75
100
125
150
175
100
200
300
400
500
600
700
800
900

Fig.5

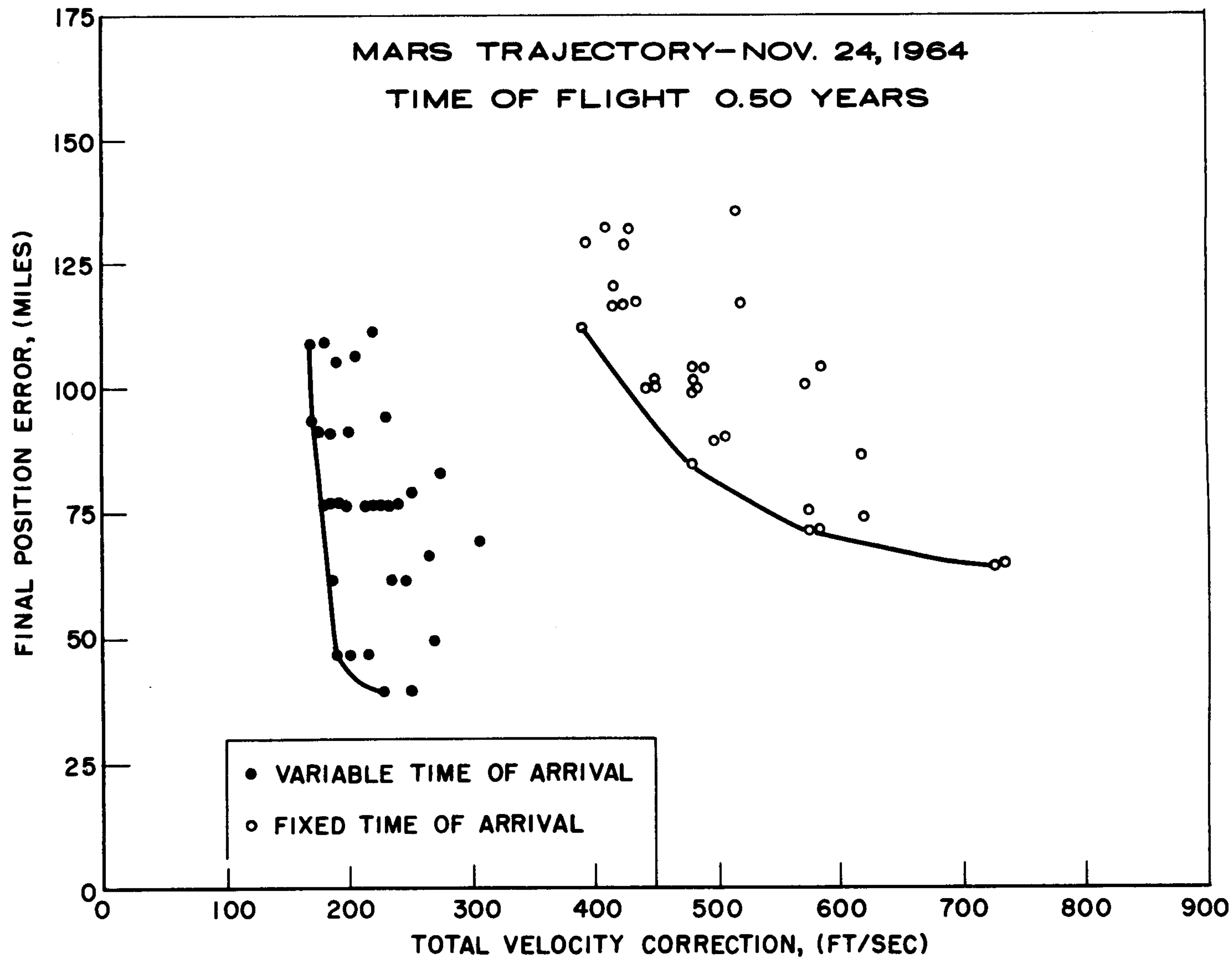
MARS TRAJECTORY—NOV. 24, 1964
TIME OF FLIGHT 0.50 YEARS
FINAL POSITION ERROR, (MILES)
175
150
125
100
75
50
25
0
0
100
200
300
400
500
600
700
800
900
TOTAL VELOCITY CORRECTION, (FT/SEC)
VARIABLE TIME OF ARRIVAL
FIXED TIME OF ARRIVAL

Fig. 6

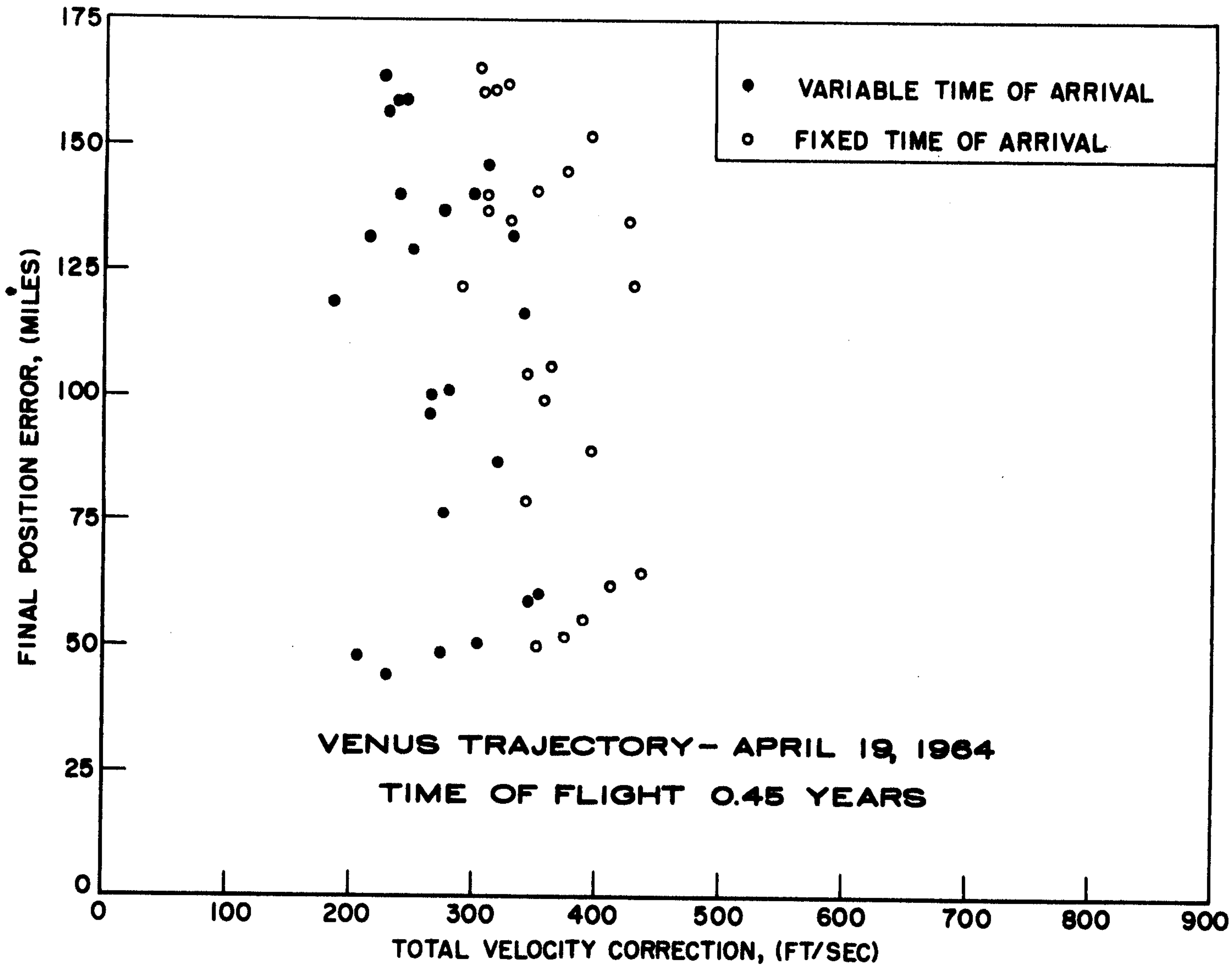
VARIABLE TIME OF ARRIVAL
FIXED TIME OF ARRIVAL
VENUS TRAJECTORY – APRIL 19, 1964
TIME OF FLIGHT 0.45 YEARS
FINAL POSITION ERROR, (MILES)
TOTAL VELOCITY CORRECTION, (FT/SEC)
175
150
125
100
75
50
25
0
100
200
300
400
500
600
700
800
900

Fig. 7

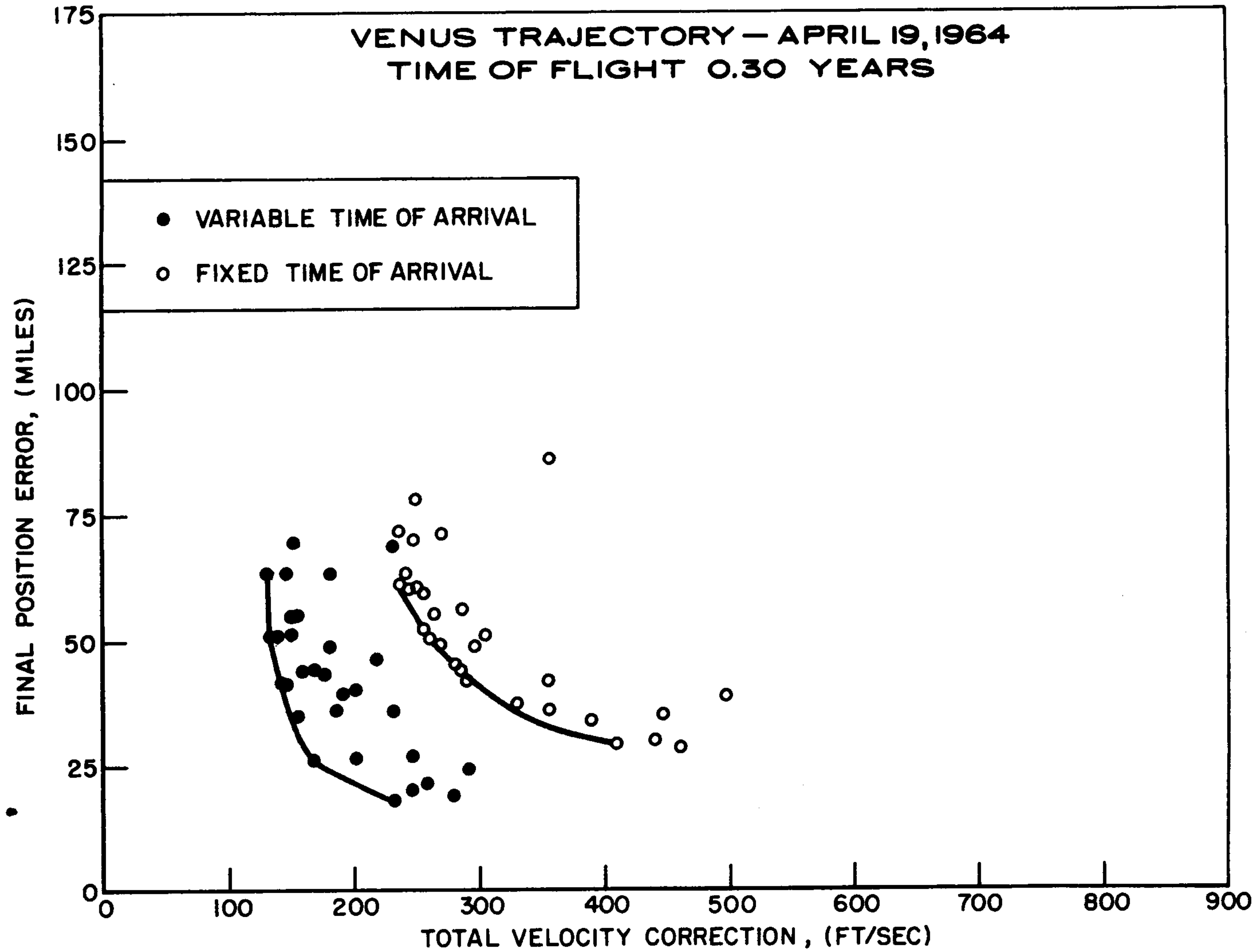
VENUS TRAJECTORY — APRIL 19, 1964
TIME OF FLIGHT 0.30 YEARS
VARIABLE TIME OF ARRIVAL
FIXED TIME OF ARRIVAL
FINAL POSITION ERROR, (MILES)
TOTAL VELOCITY CORRECTION, (FT/SEC)
175
150
125
100
75
50
25
0
0
100
200
300
400
500
600
700
800
900

Fig. 8

MARS TRAJECTORY NOV.5,1964

V_{RE} = 9968 FT/SEC V_{RM} = 9135 FT/SEC

VARIABLE TIME OF ARRIVAL NAVIGATION

TIME OF FIX	RMS VEL CORR	FINAL VEL ERROR	FINAL POS ERROR
0.001	126		
0.425	4		
0.775	14		
0.844	23		
TOTAL=	167	239	41
0.004	127		
0.375	2		
0.775	11		
0.845	31		
TOTAL=	172	240	33
0.002	126		
0.375	2		
0.775	11		
0.846	39		
TOTAL=	179	241	26
0.006	129		
0.400	3		
0.775	13		
0.847	50		
TOTAL=	195	243	19
0.005	128		
0.325	2		
0.775	13		
0.848	85		
TOTAL=	228	253	14

FIXED TIME OF ARRIVAL NAVIGATION

TIME OF FIX	RMS VEL CORR	FINAL VEL ERROR	FINAL POS ERROR
0.004	156		
0.300	7		
0.775	42		
0.843	117		
TOTAL=	322	113	61
0.002	155		
0.400	9		
0.775	30		
0.844	135		
TOTAL=	328	127	55
0.005	157		
0.300	7		
0.775	42		
0.844	132		
TOTAL=	337	127	54
0.004	156		
0.375	8		
0.775	28		
0.845	157		
TOTAL=	349	149	49
0.002	155		
0.375	8		
0.775	28		
0.846	195		
TOTAL=	385	186	44
0.006	157		
0.400	10		
0.775	30		
0.847	263		
TOTAL=	460	254	40
0.005	157		
0.325	7		
0.775	35		
0.848	401		
TOTAL=	600	392	36

TABLE 6

MARS TRAJECTORY NOV.24,1964

V_{RE} = 13526 FT/SEC V_{RM} = 25261 FT/SEC

VARIABLE TIME OF ARRIVAL NAVIGATION				FIXED TIME OF ARRIVAL NAVIGATION			
TIME OF FIX	RMS VEL CORR	FINAL VEL ERROR	FINAL POS ERROR	TIME OF FIX	RMS VEL CORR	FINAL VEL ERROR	FINAL POS ERROR
0.003	95			0.001	116		
0.025	5			0.025	6		
0.400	37			0.350	38		
0.494	34			0.494	231		
TOTAL=	170	132	93	TOTAL=	391	227	112
0.001	95			0.001	116		
0.025	5			0.100	9		
0.350	24			0.350	43		
0.494	54			0.495	276		
TOTAL=	177	135	91	TOTAL=	444	268	100
0.004	95			0.002	116		
0.025	5			0.025	6		
0.400	39			0.400	61		
0.495	40			0.496	298		
TOTAL=	179	134	77	TOTAL=	481	292	84
0.002	95			0.001	116		
0.025	5			0.025	6		
0.400	36			0.425	78		
0.496	50			0.497	374		
TOTAL=	185	136	61	TOTAL=	574	365	72
0.001	95			0.003	116		
0.025	5			0.075	9		
0.425	45			0.400	82		
0.497	51			0.498	520		
TOTAL=	196	139	46	TOTAL=	727	507	64
0.004	95						
0.025	5						
0.425	51						
0.498	75						
TOTAL=	226	152	39				

TABLE 7

VENUS TRAJECTORY APRIL 19,1964

V_{RE} = 9688 FT/SEC V_{RV} = 18357 FT/SEC

VARIABLE TIME OF ARRIVAL NAVIGATION				FIXED TIME OF ARRIVAL NAVIGATION			
TIME OF FIX	RMS VEL CORR	FINAL VEL ERROR	FINAL POS ERROR	TIME OF FIX	RMS VEL CORR	FINAL VEL ERROR	FINAL POS ERROR
0.006	131			0.006	161		
0.225	5			0.225	7		
0.400	22			0.400	26		
0.443	28			0.443	98		
TOTAL=	186	125	117	TOTAL=	292	92	122
0.002	129			0.002	159		
0.200	6			0.200	8		
0.400	38			0.400	44		
0.447	57			0.447	140		
TOTAL=	231	137	44	TOTAL=	350	133	49

TABLE 8

VENUS TRAJECTORY APRIL 19,1964

V_{RE} = 14206 FT/SEC V_{RV} = 13339 FT/SEC

VARIABLE TIME OF ARRIVAL NAVIGATION				FIXED TIME OF ARRIVAL NAVIGATION			
TIME OF FIX	RMS VEL CORR	FINAL VEL ERROR	FINAL POS ERROR	TIME OF FIX	RMS VEL CORR	FINAL VEL ERROR	FINAL POS ERROR
0.003	92			0.004	112		
0.175	5			0.100	6		
0.250	7			0.225	23		
0.293	24			0.294	98		
TOTAL=	128	116	63	TOTAL=	238	89	61
0.002	92			0.004	112		
0.150	4			0.125	5		
0.250	9			0.250	23		
0.294	28			0.295	119		
TOTAL=	133	117	51	TOTAL=	259	108	50
0.001	91			0.001	111		
0.150	5			0.100	6		
0.250	9			0.200	17		
0.295	34			0.296	144		
TOTAL=	139	118	41	TOTAL=	279	136	45
0.005	93			0.001	111		
0.125	5			0.075	8		
0.250	14			0.225	38		
0.296	42			0.296	131		
TOTAL=	154	120	35	TOTAL=	288	121	42
0.006	93			0.001	111		
0.150	6			0.075	8		
0.250	9			0.225	38		
0.297	56			0.298	252		
TOTAL=	164	125	26	TOTAL=	409	241	29
0.003	92			0.001	111		
0.075	8			0.025	13		
0.250	37			0.225	97		
0.298	90			0.298	237		
TOTAL=	227	139	18	TOTAL=	459	229	28

TABLE 9

REFERENCES

1. Battin, Richard H. , The Determination of Round-Trip Planetary Reconnaissance Trajectories, Journal of the Aero-Space Sciences, Sept. 1959.
2. Battin, Richard H. , and Laning, J. H. Jr. , A Navigation Theory for Round-Trip Reconnaissance Missions to Venus and Mars, Proceedings of the Fourth AFBMD/STL Symposium on Ballistic Missile and Space Technology, Pergamon Press, 1960.